Games, graphs, and machines

- Read wattle.
- Watch pre lec videos & do reading
- Join Zulip

1.7 Exercises

1.7.1 Write down the cardinality of the following sets:

1. {} = ϕ size 0
2. {{}}
3. {{},{{}}}

{ ↷ elements sep by commas }

1. No elements
2. The only elt is {} ⇒ 1
3. Elts : ϕ , {ϕ} ⇒ 2
4. { {}, {{}, {{{}}}} }
   elt 1 elt 2

1.7 Exercises

1.7.1 Write down the cardinality of the following sets:

1. $\{\}$
2. $\{\{\}\}$
3. $\{\{\}, \{\{\}\}\}$

1.7.2 True or false?

True ✓ 1. $\{\} \subset \{\}$ anything

False 2. $\{\} \in \{\}$ anything

3. $\{\} \in \{\{\}\}$

$\subset$ = is a subset of

$\in$ = is an elt of

$\{\} \subset \{\{\}\}$ } both true.

1.7.3 The cardinality of the power set

What is the size of the power set of $\{1, 2, 3, 4\}$? 2^4
What about $\{1, 2, 3, 4, 5\}$? 2^5
What about $\{1, \ldots, 100\}$? 2^{100}

$\text{Pow}(\{1,2,3,4\}) = \{\emptyset, \{1\}, \{2\}, \{3\}, \{4\}, \{1,2\}, \{1,3\}, \{1,4\}, \ldots, \{1,2,3,4\}\}$

subsets of $\{1,2,3,4\}$

	1	2	3	4 ... n	✓ or ✗
$\emptyset$:	✗	✗	✗	✗	for each pos.
$\{1\}$:	✓	✗	✗	✗	⇒ Total
$\{2,3\}$:	✗	✓	✓	✗	$\underbrace{2 \times 2 \times 2 \cdots \times 2}_{n}$
$\{1,2,3,4\}$:	✓	✓	✓	✓	

$\Rightarrow 2^n$ subsets of $\{1, 2, \ldots, n\}$.

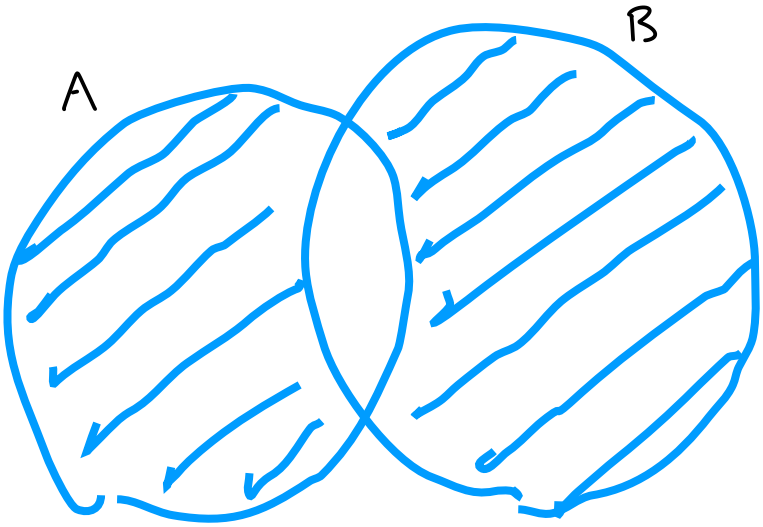
1.7.4 The cardinality of the power set (continued)

Suppose the size of A is n. What is the size of the power set of A? Why? 2^n

1.7.5 The symmetric difference

Suppose A and B are represented by the circles below. Using the operations of union, intersection, and difference, express the shaded set.

$\cup$, $\cap$, $-$



$A \cup B - A \cap B$

$= (A-B) \cup (B-A)$

1.7.6 Set operations

Let $A = \{x^2 \mid x \in \mathbf{Z}\}$ and $B = \{x^3 \mid x \in \mathbf{Z}\}$. Write the smallest 5 elements of

1. $A \cup B$
2. $A \cap B$
3. $A - B$
4. $B - A$

1.7.7 Set operations (continued)

Is the following true or false: $|A - B| = |A| - |B|$.
If it is true, explain why.
If it is not true, give a counter-example.

$A = \{1\}$ $B = \{2\}$

$A - B = \{1\}$

If $B \subset A$ then $|A-B| = |A| - |B|$

Similarly $|A \cup B|$ not alw $|A| + |B|$